Some Additional Notes on First Order Logic

Amir Aavani Amir@Aavani.net
http://Amir.Aavani.net/CS/AI-86

16. Dezember 2007

Kinship domain

1. $\forall m, c \;\; Mother(c) = m \Leftrightarrow Female(m) \land Parent(m, c)$

Kinship domain

1. $\forall m, c \;\; Mother(c) = m \Leftrightarrow Female(m) \wedge Parent(m, c)$
2. $\forall d, h \;\; Husband(h, w) \Leftrightarrow Male(h) \wedge Spouse(h, w)$

Kinship domain

1. $\forall m, c \;\; Mother(c) = m \Leftrightarrow Female(m) \wedge Parent(m, c)$
2. $\forall d, h \;\; Husband(h, w) \Leftrightarrow Male(h) \wedge Spouse(h, w)$
3. $\forall x \;\; Male(x) \Leftrightarrow \neg Female(x)$

Kinship domain

1. $\forall m, c \;\; Mother(c) = m \Leftrightarrow Female(m) \wedge Parent(m, c)$
2. $\forall d, h \;\; Husband(h, w) \Leftrightarrow Male(h) \wedge Spouse(h, w)$
3. $\forall x \;\; Male(x) \Leftrightarrow \neg Female(x)$
4. $\forall p, c \;\; Parent(p, c) \Leftrightarrow Child(c, p)$

Kinship domain

1. $\forall m, c \;\; Mother(c) = m \Leftrightarrow Female(m) \wedge Parent(m, c)$
2. $\forall d, h \;\; Husband(h, w) \Leftrightarrow Male(h) \wedge Spouse(h, w)$
3. $\forall x \;\; Male(x) \Leftrightarrow \neg Female(x)$
4. $\forall p, c \;\; Parent(p, c) \Leftrightarrow Child(c, p)$
5. $\forall g, c \;\; Grandparent(g, c) \Leftrightarrow \exists p \;\; Parent(g, p) \wedge Parent(p, c)$

Kinship domain

1. $\forall m, c \;\; Mother(c) = m \Leftrightarrow Female(m) \wedge Parent(m, c)$
2. $\forall d, h \;\; Husband(h, w) \Leftrightarrow Male(h) \wedge Spouse(h, w)$
3. $\forall x \;\; Male(x) \Leftrightarrow \neg Female(x)$
4. $\forall p, c \;\; Parent(p, c) \Leftrightarrow Child(c, p)$
5. $\forall g, c \;\; Grandparent(g, c) \Leftrightarrow \exists p \;\; Parent(g, p) \wedge Parent(p, c)$
6. $\forall x, y \;\; Sibling(x, y) \Leftrightarrow x \neq y \wedge \exists p \;\; Parent(p, x) \wedge Parent(p, y)$

Axioms and Theorem

1. Axioms provide basic information about domain.

Axioms and Theorem

1. Axioms provide basic information about domain.
2. Not all statements about a domain are axioms. Some are theorems.

Axioms and Theorem

1. Axioms provide basic information about domain.
2. Not all statements about a domain are axioms. Some are theorems.
3. Theorems can be entailed from axioms and other theorems.

Axioms and Theorem

1. Axioms provide basic information about domain.
2. Not all statements about a domain are axioms. Some are theorems.
3. Theorems can be entailed from axioms and other theorems.
4. Example: $\forall x, y \ Sibling(x, y) \Leftrightarrow Sibling(y, x)$

Axioms and Theorem

1. Axioms provide basic information about domain.
2. Not all statements about a domain are axioms. Some are theorems.
3. Theorems can be entailed from axioms and other theorems.
4. Example: $\forall x, y \; Sibling(x, y) \Leftrightarrow Sibling(y, x)$
5. Is it a theorem?:
   $Male(Gorge) \wedge Spouse(Gorge, Laura) \Rightarrow Female(Laura)$

Axioms and Theorem, Cont

1. Not all axioms are definitions.
2. $\forall x \; Person(x) \; \Leftrightarrow \cdots$

Axioms and Theorem, Cont

1. Not all axioms are definitions.
2. $\forall x \ Person(x) \ \Leftrightarrow \cdots$
3. Some provide more general information about certain predicates with out constituting a definition
4. $\forall x \ Person(x) \ \Rightarrow \cdots$
5. $\forall x \ \cdots \Rightarrow Person(x)$

Definition of Sets in First Order Logic

1. $\forall s\ Set(s) \Leftrightarrow (S = \{\}) \vee (\exists x, s_2\ Set(s_2) \wedge s = \{x|s_2\})$

Definition of Sets in First Order Logic

1. $\forall s\ Set(s) \Leftrightarrow (S = \{\}) \vee (\exists x, s_2\ Set(s_2) \wedge s = \{x|s_2\})$
2. $\neg\exists x, s\{x|s\} = \{\}$

Definition of Sets in First Order Logic

1. $\forall s \; Set(s) \Leftrightarrow (S = \{\}) \vee (\exists x, s_2 \; Set(s_2) \wedge s = \{x|s_2\})$
2. $\neg\exists x, s\{x|s\} = \{\}$
3. $\forall x, s \; x \in s \Leftrightarrow s = \{x|s\}$

Definition of Sets in First Order Logic

1. $\forall s \; Set(s) \Leftrightarrow (S = \{\}) \lor (\exists x, s_2 \; Set(s_2) \land s = \{x|s_2\})$
2. $\neg\exists x, s\{x|s\} = \{\}$
3. $\forall x, s \; x \in s \Leftrightarrow s = \{x|s\}$
4. $\forall x, s \; x \in s \Leftrightarrow$

Definition of Sets in First Order Logic

1. $\forall s \; Set(s) \Leftrightarrow (S = \{\}) \vee (\exists x, s_2 \; Set(s_2) \wedge s = \{x|s_2\})$
2. $\neg \exists x, s\{x|s\} = \{\}$
3. $\forall x, s \; x \in s \Leftrightarrow s = \{x|s\}$
4. $\forall x, s \; x \in s \Leftrightarrow \exists y, s_2 (s = \{y|s_2\} \wedge$

Definition of Sets in First Order Logic

1. $\forall s\ Set(s) \Leftrightarrow (S = \{\}) \vee (\exists x, s_2\ Set(s_2) \wedge s = \{x|s_2\})$
2. $\neg \exists x, s\{x|s\} = \{\}$
3. $\forall x, s\ x \in s \Leftrightarrow s = \{x|s\}$
4. $\forall x, s\ x \in s \Leftrightarrow \exists y, s_2(s = \{y|s_2\} \wedge (x = y) \vee (x \in s_2))$

Definition of Sets in First Order Logic

1. $\forall s\ Set(s) \Leftrightarrow (S = \{\}) \vee (\exists x, s_2\ Set(s_2) \wedge s = \{x|s_2\})$
2. $\neg\exists x, s\{x|s\} = \{\}$
3. $\forall x, s\ x \in s \Leftrightarrow s = \{x|s\}$
4. $\forall x, s\ x \in s \Leftrightarrow \exists y, s_2(s = \{y|s_2\} \wedge (x = y) \vee (x \in s_2))$
5. $\forall s_1, s_2\ s_1 \in s_2 \Leftrightarrow (\forall x \in s_1 \Rightarrow x \in s_2)$

Definition of Lists in First Order Logic

1. Your exercise!!