

Artificial Intelligence- Logical Agents

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Outline

① Knowledge-based Agents

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- 1 Knowledge-based Agents
- 2 Wampus world

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- 3 Logic in general
- 4 Propositional Logic

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- 1 **Knowledge Base(KB)**: Set of sentences in knowledge representation language.
- 2 Each sentence represent some assertion about the world.
- 3 **Tell** method is a way to add new statements to KB.
- 4 **Ask** method is a way to query from KB.
- 5 **Inference**: deriving new sentences from old, using logic.

Knowledge-Based Agents, Cont

```
function KB-AGENT (percept) returns an action
  static KB, a knowledge base
    t, a counter initially 0

  TELL (KB,MAKE-PERCEPT-STATEMENT(percept,t))
  action= ASK(KB,MAKE-ACTION-QUERY(~))
  TELL (KB,MAKE-ACTION-SENTENCE(action,t))
  t= t+ 1
  return action
```

Knowledge-Based Agents, Cont

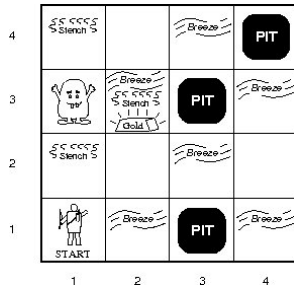
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Knowledge-Based Agents, Cont

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- 2 In **Implementation Level**:, one talks about data structures and algorithms used to work with statements.
















Wampus world, PEAS

- 1 Performance measure:
 - . gold: +1000, death: -1000
 - . -1 per step, -10 for using arrow.
- 2 Environment:
 - . A 4x4 grid of rooms.
 - . Agent starts in square [1,1].
 - . There may be a pit in each square.
 - . etc.
- 3 Actuators: Turn left, Turn right, Forward
 - . Forward, Grab, Release, Shoot
- 4 Sensors: Stench, Breeze, Glitter, Bump, Scream



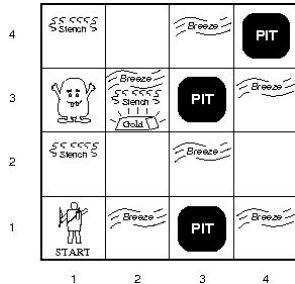
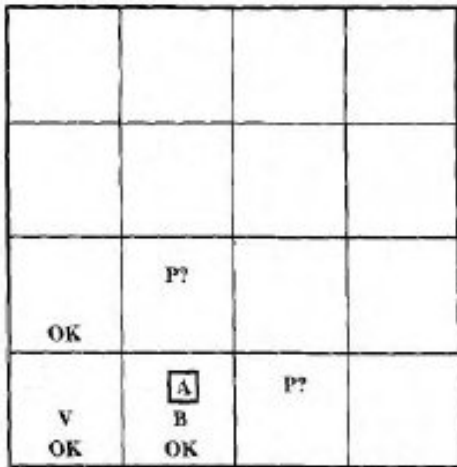
Wampus world, exploration

OK			
OK <div>A</div>	OK		

4	 Stench		 Breeze	 PIT
3	  Breeze  Stench  Gold	 PIT	 Breeze	
2	 Stench		 Breeze	
1	 START	 Breeze	 PIT	 Breeze
	1	2	3	4

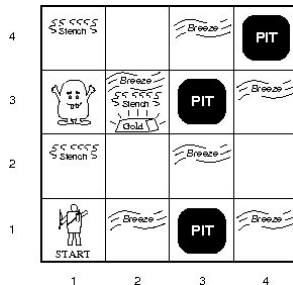
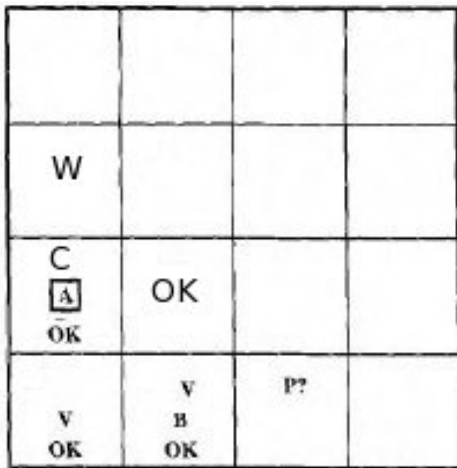
A: [-,-,-,-]

Wampus world, exploration



B: [-,B,-,-]

Wampus world, exploration



C: [S,-,-,-]

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 - . Assuming pits uniformly distributed,
 - . (2,2) has pit with prob. 0.86.
- ② Consider if we have Smell in (1,1)

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 - . Wampus wasn't there \Rightarrow safe

Logic in general

- 1 **Logics** are formal languages for representing information.
- 2 To define a logic, its **syntax** and **semantics** should be defined.

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- 3 Syntax defines the structure of sentences.
- 4 Semantic defines the meaning of sentences.

Entailment

- 1 A **Entails** B (informally) means that B follows logically from B.
- 2 $KB \models \alpha$ (Knowledge base KB entails α)
 - . if and only if
 - α is true in all worlds(models) where KB is true.

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 - α is true in all worlds(models) where KB is true.
- ③ $KB = \{\text{It is cold, It is raining}\} \models \text{It is cold and rainy.}$

Models

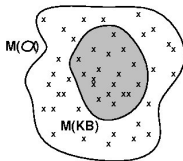
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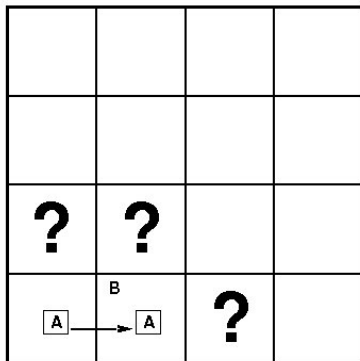
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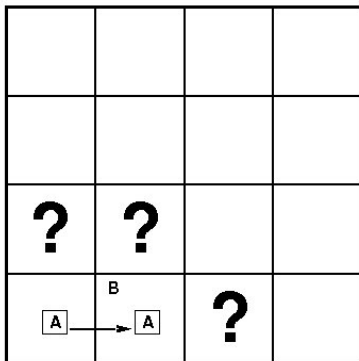


Wampus models



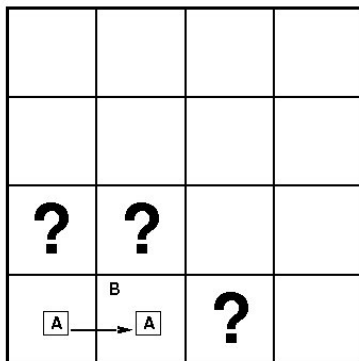
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Wampus models



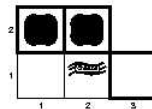
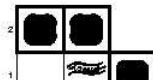
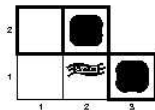
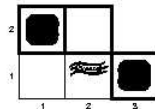
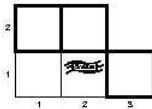
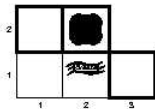
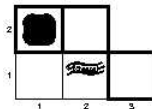
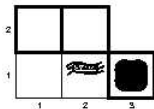
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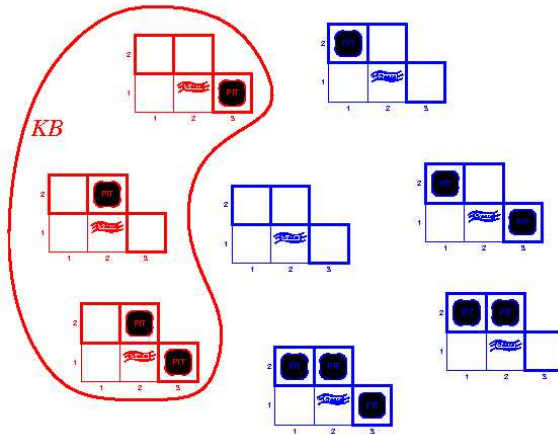


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- 3 There are 8 modals.

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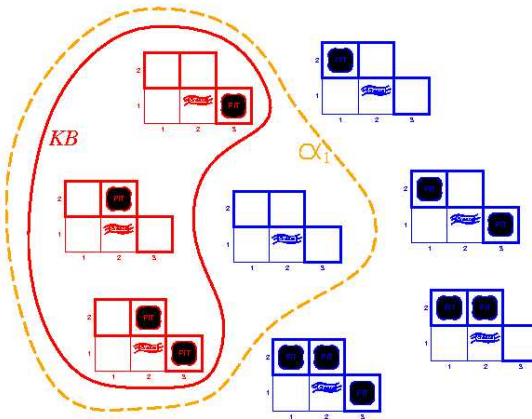


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KB = wampus-world rules + observations

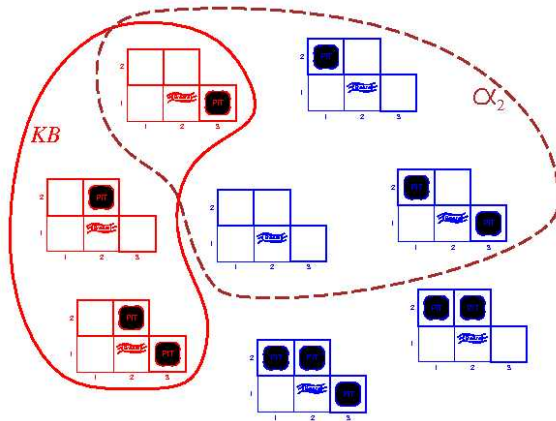
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$\alpha_1 = [1, 2]$ is safe

Wampus models, Cont



KB = wampus-world rules + observations
 $\alpha_2 = [2,2]$ is safe.

Inference

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- ① **Soundness**= Truth preserving.
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- ③ A **Complete** and **Sound** inference procedure will answer any question whose answer follows from what is known by the *KB*.

Propositional logic- Syntax

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- ② $\text{AtomicSentence} \rightarrow \text{True} \mid \text{False} \mid \text{Symbol}$
- ③ $\text{Symbol} \rightarrow P \mid Q \mid R \mid \dots$
- ④ $\text{ComplexSentence} \rightarrow \neg \text{Sentence}$
- ⑤ $\text{ComplexSentence} \rightarrow (\text{Sentence} \wedge \text{Sentence})$

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- 6 $S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true and $S_1 \Rightarrow S_2$ is true.

Wampus world with propositional logic

- 1 $P_{i,j}$ is true if there is a pit in $[i,j]$.

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- 4 $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
- 5 $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

Wampus world with propositional logic, Cont

① $R_1 : \neg P_{1,1}$

Wampus world with propositional logic, Cont

1 $R_1 : \neg P_{1,1}$

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③ $R_3 : B_{2,1}$

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- 3 $R_3 : B_{2,1}$
- 4 $R_4 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

Wampus world with propositional logic, Cont

- ❶ $R_1 : \neg P_{1,1}$
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- ❹ $R_4 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
- ❺ $R_5 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

Inference by enumeration

```
function TT-ENTAILS? (KB, a) returns true or false
  symbols= list of the symbols in KB and a.
  return TT-CHECK-ALL (KB, a, symbols, []);
```

```
function TT-CHECK-ALL (KB, a, symbols, models)
  returns true or false
  if Empty(symbols) then
    if PL-TRUE?(KB,models) then return PL-TRUE?(a,models)
    else return false;
  else
    P= FIRST(symbols); rest= REST(symbols);
    return TT-CHECK-ALL(KB,a,rest,EXTEND(P,true,modal))
      and TT-CHECK-ALL(KB,a,rest,EXTEND(P,false,modal))
```


Some concepts in propositional logic

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- ⑨ $(a \wedge (b \vee c)) \equiv (a \wedge b) \vee (a \wedge c)$ distributivity of \wedge over \vee
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Reasoning patterns in propositional logic

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5 And-Elimination:
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- 2 Finding a proof is a **search problem**.
- 3 Problem Statement: ?
- 4 Goal Statement: ?
- 5 Successor Function: ?
- 6 Path Cost: ?

Reasoning patterns in propositional logic, Cont

- 1 A **Horn clause**: is disjunction of literals of which atmost one is positive.
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- 3 There are two algorithms to work with Horn clauses(forward checking and backward checking).
- 4 Deciding entailment with Horn clauses can be done in **linear time**.

Forward chaining

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Forward Chaining, Cont

function FC-ENTAILS?(KB, q) returns true or false

```
while agenda is not empty do
  p= POP(agenda)
  unless inferred [p] do
    inferred [p]= true
    foreach c in whose promises p appears do
      Dec (count[c])
      if count[c]= 0 then
        if HEAD[c]=q then
          return true;
        push(HEAD[c],agenda);
  return false
```

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- 4 Resolution is sound and complete for propositional logic.

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- 3 If an empty clause generated, returns true.

Resolution, Cont

```
function Resolution?(KB, q) returns true or false

  clauses= set of clauses in the CNF format of KB^ not a
  loop
  new= {}
  for each Ci, Cj in clauses do
    resolvants=RESOLVE(Ci,Cj);
    if resolvants contains empty clauses then
      return true
    new= new U resolvants
  if new is a subset of clauses return false;
  clauses= clauses U new
```

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- 3 Resolution+ complete search algorithm: complete inference algorithm.

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 - Backtracking search
 - Local search

Complete backtracking search (Davis-Putnam)

```
function DPLL(clauses,symbols,model) return true or false
  if every clauses is true in model return true
  if some clauses is false in model return false
  P,v=FIND-PURE-SYMBOL(symbols,clauses,model)
  if P!=null return DPLL(clauses,symbols-P,model+(P,v))
  P,value= FIND-UNIT-SYMBOL(clauses,model)
  if P!=null return DPLL(clauses,symbols-P,model+(P,v))
  P=FIRST(symbols);rest=REST(symbols);
  return DPLL(clauses,rest,model+(P,true)) or
         DPLL(clauses,rest,model+(P,false))
```

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- 4 Unfortunately, **min-conflict** has many local minima.
- 5 Various forms of randomness are required.

Local-search algorithm- Cont

```
function WALKSAT(clauses,p,max-flips)returns a model or fail
model=random assignment of true/false to symbols in clause
for i:= 1 to max-flip do
  if model satisfies clauses then return model
  clause= a randomly selected clause
           that is false in model.
  with prob. p:
    flip value of randomly selected symbol from clause
  else
    flip symbol in clause which maximizes no. of
    satisfied clauses.
```

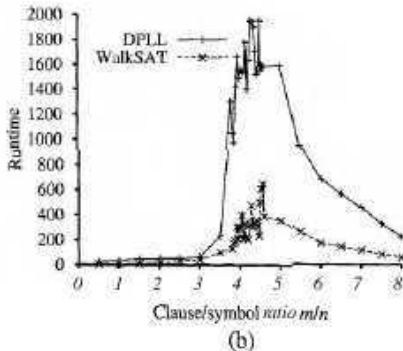
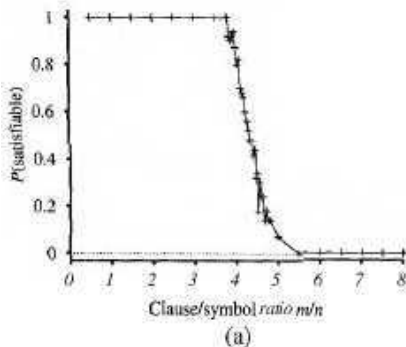

Local-search algorithm, Cont

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- 1 WALKSAT may returns a model if one exists.
- 2 It **can not tell** whether the statement is unsatisfiable.
- 3 But, WALKSAT can tell us a statement my be unsatisfiable.
- 4 Satisfiability may be a hard problem (depending on $\frac{\text{No. of symbols}}{\text{No. of clauses}}$)

Local-search algorithm, Cont



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⑤ Problem has 64 symbols and 155 statements.

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